

A Calculation on the Self-field of a Point Charge and the Unruh Effect

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Abstract

Within the context of quantum field theory in curved spacetimes, Hacyan and Sarmiento defined the vacuum stress-energy tensor with respect to the accelerated observer. They calculated it for uniform acceleration and circular motion, and derived that the rotating observer perceives a flux. Mane related the flux to synchrotron radiation. In order to investigate the relation between the vacuum stress and bremsstrahlung, we estimate the stress-energy tensor of the electromagnetic field generated by a point charge, at the position of the charge. We use the retarded field as a self-field of the point charge. Therefore the tensor diverges if we evaluate it as it is. Hence we remove the divergent contributions by using the expansion of the tensor in powers of the distance from the point charge. Finally, we take an average for the angular dependence of the expansion. We calculate it for the case of uniform acceleration and circular motion, and it is found that the order of the vacuum stress multiplied by $\pi\alpha$ ($\alpha = e^2/\hbar c$ is the fine structure constant) is equal to that of the self-stress. In the Appendix, we give another trial approach with a similar result.

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1 Introduction

In Minkowski spacetime, a field is quantized with respect not only to the inertial frame, but also to a uniformly accelerated frame [1]. Definitions of the vacua of these quantizations are not equivalent. In the quantization with respect to the uniformly accelerated frame, the vacuum of the inertial frame corresponds to a thermal bath in which the temperature is proportional to the acceleration of the accelerated frame. Therefore, one can interpret that a uniformly accelerated observer in the vacuum perceives a thermal bath of temperature proportional to his acceleration [2, 3]. This is referred to as the Unruh effect. This interpretation is confirmed by using the Unruh-DeWitt detector, which is a mathematically idealized detector of the field quanta [3, 4, 5, 6]. It is well known that, when the detector is uniformly accelerated, the transition probability between the internal states of the detector indicate the thermal behavior. The detector is also excited in any accelerated motion, but, in general, the transition probability does not indicate the thermal behavior. The behavior of the detector in a rotating orbit (circular Unruh effect [7]) is particularly interesting because of the possibility of experimental verification [8].

It is interesting to conjecture how an accelerated electron can be affected by the Unruh-like effect ascribed above. The spin of the electron could correspond to the internal degree of freedom of the detector [8], and it is concerned with the experimental verificaton of the circular Unruh effect. On the other hand, it would be also interesting to investigate the relation between bremsstrahlung and Unruh-like effect [9, 10, 11, 12, 13], and this is what we investigate in this paper. In this connection, there is a long-standing problem of classical electrodynamics concerning whether a uniformly accelerated electron radiates [14, 15, 16], and it would be interesting to consider this problem in connection with the Unruh effect. Discussions from this point of view are found, for example, in Refs. [9, 12] and [17].

In this paper, we make a calculation within the classical theory [18], concerned with the discussion of Mane [10], which relates bremsstrahlung to the Unruh effect. The outline of the discussion is the following. Within quantum field theory in curved spacetimes, Hacyan and Sarmiento defined the spectrum of the stress-energy tensor of the electromagnetic vacuum with respect to an accelerated observer, and calculated it for uniformly accelerated motion [19] and circular motion [20]. For uniform acceleration, they obtained a spectrum of an isotropic thermal bath, and for circular motion, they derived that the spectrum is not thermal, and there is a flux directed along the tangent velocity of the observer. Hacyan and Sarmiento pointed out the possibility that the flux would cause some friction-like effect on a rotating particle. Mane suggested that this friction-like effect is related to synchrotron radiation. Mane discussed that if the flux is coupled to an electron through the fine structure constant $\alpha = e^2/\hbar c$, the order of energy loss of the electron is classical, and it corresponds to the order derived from the Larmor formula.

We are interested in how the vacuum stress can be related to the classical bremsstrahlung,

and we propose to evaluate the stress-energy tensor of the electromagnetic field generated by a point charge, at the position of the charge [21]. (We will call this quantity the self-stress only for simplicity, although this term is generally used with a different meaning and context. See Section 17.5 of Ref. [24].) We use the retarded field as a self-field, and thus the tensor diverges if we evaluate it as it is. We consider the expansion of the tensor in powers of the distance from the point charge, and we remove the divergent contributions in the limit that the distance approaches zero. That is, we regard the renormalized tensor as the terms of zero-th order in the expansion. Although the result depends on the direction along which we take the limit, we remove the directional dependence by taking an angular average. (In 1971, Teitelboim showed that the radiation reaction force of the Lorentz-Dirac equation can be obtained by averaging the retarded field around a point charge [22]. Our method of averaging the retarded stress-energy tensor is the same as his method of averaging the retarded field.) We calculate this average for uniform acceleration and circular motion, and it is found that the order of the vacuum stress multiplied by $\pi\alpha$ is equivalent to that of the self-stress which we calculate. In Appendix B, we give an alternative evaluation of the self-stress in which we use the expansion of the retarded field in powers of the retarded time, and obtain a similar result.

In section 2, we review the vacuum stress-energy tensor defined by Hacyan and Sarmiento and the discussion given by Mane. In section 3, We expand the retarded field by using the method of Dirac [23] and briefly discuss the work of Teitelboim. The expansion is used to evaluate the zero-th terms in the expansion of the stress-energy tensor of the self-field in section 4. The result is discussed in section 5. Throughout the paper, we use Gaussian units and the metric with signature $(+, -, -, -)$. We employ natural units in which $c = \hbar = 1$, and we write c and \hbar explicitly only when an order estimation is needed.

2 Vacuum stress and the discussion of Mane

2.1 Vacuum stress-energy tensor

Within the context of quantum field theory in curved spacetimes, Hacyan and Sarmiento defined the electromagnetic vacuum stress-energy tensor with respect to the accelerated observer [19, 20]. Let us review their work, mainly focusing on the points concerned with our problem. The expectation value of the stress-energy tensor of the electromagnetic field is

$$T_{\mu\nu} = \frac{1}{16\pi} \lim_{x' \rightarrow x} \langle 0_M | 4F_{(\mu}^\alpha(x)F_{\nu)\alpha}(x') + \eta_{\mu\nu} F_{\lambda\beta}(x)F^{\lambda\beta}(x') | 0_M \rangle, \quad (1)$$

where $|0_M\rangle$ represents the Minkowski vacuum. We define

$$D_{\mu\nu}^+(x, x') \equiv \frac{1}{4} \langle 0_M | 4F_{(\mu}^\alpha(x)F_{\nu)\alpha}(x') + \eta_{\mu\nu} F_{\lambda\beta}(x)F^{\lambda\beta}(x') | 0_M \rangle,$$

$$D_{\mu\nu}^-(x, x') \equiv D_{\mu\nu}^+(x', x), \quad (2)$$

so that

$$T_{\mu\nu} = \frac{1}{4\pi} \lim_{x' \rightarrow x} D_{\mu\nu}^\pm(x, x'). \quad (3)$$

The decomposition of $F_{\mu\nu}$ into the destruction and creation operators and the action of these operators on $|0_M\rangle$ lead to the relation

$$\langle 0_M | F_{(\mu}^\alpha(x) F_{\nu)\alpha}(x') | 0_M \rangle = 8\pi \partial_\mu \partial_{\nu'} D^\pm(x, x'), \quad (4)$$

where

$$D^\pm(x, x') = -\frac{1}{4\pi^2} \frac{1}{(t - t' \mp i\epsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2} \quad (5)$$

are the Wightman functions for the massless scalar field. By performing the differentiations, we find that

$$\begin{aligned} & \langle 0_M | F_{(\mu}^\alpha(x) F_{\nu)\alpha}(x') | 0_M \rangle \\ &= \frac{4}{\pi} \frac{4(x_\mu - x'_\mu)(x_\nu - x'_\nu) - \eta_{\mu\nu}(x_\alpha - x'_\alpha)(x^\alpha - x'^\alpha)}{[(t - t' \mp i\epsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2]^3}. \end{aligned} \quad (6)$$

The contraction of the indices of this equation gives

$$\langle 0_M | F_{\lambda\beta}(x) F^{\lambda\beta}(x') | 0_M \rangle = 0, \quad (7)$$

which leads to

$$T_{\mu\nu} = \frac{1}{4\pi} \lim_{x' \rightarrow x} \langle 0_M | F_{(\mu}^\alpha(x) F_{\nu)\alpha}(x') | 0_M \rangle. \quad (8)$$

That is, we find that the second term on the right-hand side of Eq. (1) do not contribute to $T_{\mu\nu}$.

Now let us evaluate the spectrum of the stress-energy tensor detected by an observer through the world line

$$z^\alpha = z^\alpha(\tau), \quad (9)$$

where τ is the proper time of the detector. It follows that

$$\begin{aligned} T_{\mu\nu}[z^\alpha(\tau)] &= \frac{1}{4\pi} \int_{-\infty}^{\infty} d\sigma \delta(\sigma) D_{\mu\nu}^+(\tau + \sigma/2, \tau - \sigma/2) \\ &= \frac{1}{8\pi^2} \int_{-\infty}^{\infty} d\sigma \int_0^{\infty} d\omega e^{i\omega\sigma} [D_{\mu\nu}^+(\tau + \sigma/2, \tau - \sigma/2) + D_{\mu\nu}^-(\tau + \sigma/2, \tau - \sigma/2)], \end{aligned} \quad (10)$$

where

$$D_{\mu\nu}^\pm(\tau + \sigma/2, \tau - \sigma/2) \equiv D_{\mu\nu}^\pm(z(\tau + \sigma/2), z(\tau - \sigma/2)). \quad (11)$$

Using the Fourier transform

$$\tilde{D}_{\mu\nu}^\pm(\tau, \omega) = \int_{-\infty}^{\infty} d\sigma e^{i\omega\sigma} D_{\mu\nu}^\pm(\tau + \sigma/2, \tau - \sigma/2), \quad (12)$$

the stress-energy tensor can be written as

$$T_{\mu\nu}[z^\alpha(\tau)] = \frac{1}{8\pi^2} \int_0^\infty [\tilde{D}_{\mu\nu}^+(\tau, \omega) + \tilde{D}_{\mu\nu}^-(\tau, \omega)] d\omega. \quad (13)$$

Because the $D_{\mu\nu}^\pm(\tau + \sigma/2, \tau - \sigma/2)$ are even functions with respect to σ , and because of Eqs. (2), (6) and (7), we can write

$$D_{\mu\nu}^\pm(\tau + \sigma/2, \tau - \sigma/2) = \frac{A_{\mu\nu}}{(\sigma \mp i\epsilon)^4} - \frac{B_{\mu\nu}}{(\sigma \mp i\epsilon)^2} + D_{\mu\nu}(\tau, \sigma), \quad (14)$$

where $A_{\mu\nu}$ and $B_{\mu\nu}$ are functions of τ , and $D_{\mu\nu}(\tau, \sigma)$ is by definition free of poles at $\sigma = \pm i\epsilon$. Inserting Eq. (14) into Eq. (13), we obtain that

$$T_{\mu\nu}[z^\alpha(\tau)] = \frac{1}{8\pi^2} \int_0^\infty \left[\frac{A_{\mu\nu}}{3} \omega^3 + 2B_{\mu\nu}\omega \right] d\omega + \frac{1}{4\pi} D_{\mu\nu}(\tau, 0). \quad (15)$$

One can interpret this equation as expressing that the divergent integral term corresponds to the zero-point energy, and the last term gives the physically observable stress-energy tensor.

We can use Eq. (13) to obtain the spectrum of the stress-energy tensor. For example, for uniform acceleration with acceleration a , Eq. (13) turns out to be

$$T_{\mu\nu} = \frac{1}{3\pi^2} (4u_\mu u_\nu - \eta_{\mu\nu}) \int_0^\infty \omega (\omega^2 + a^2) \left[\frac{1}{2} + \frac{1}{e^{2\pi\omega/a} - 1} \right] d\omega, \quad (16)$$

where u_μ is the 4-velocity of the observer. If one considers that the effect of the acceleration changes the density of states from $\omega^2 d\omega$ to $(\omega^2 + a^2) d\omega$ [This change is clarified by the spin of the field. see Ref. [19].], the above spectrum can be interpreted as a Planck spectrum. The term $\omega(\omega^2 + a^2)/2$ is considered as the zero-point energy of the field in the accelerated frame, and this just corresponds to the divergent term of Eq. (15).

Hacyan and Sarmiento pointed out that one can remove the divergent contribution of the vacuum stress-energy tensor by moving the pole of the Wightman function properly. But they judged that this is a rather *ad hoc* procedure, and they decided not to discard the zero-point energy in Eq. (15) (see section V of Ref. [20]). Thus they were careful with divergence elimination. However, they regarded virtually

$(4\pi)^{-1}D_{\mu\nu}(\tau, 0)$ as the renormalized stress-energy tensor. Therefore we also adopt it as the renormalized vacuum stress-energy tensor.

Here we give a simple interpretation of their renormalization procedure to contrast it with our renormalization procedure of the classical self-stress, which will be introduced later. The interpretation is as follows. One first expands $T_{\mu\nu}$ in powers of the proper time as in Eq. (14), and then removes contributions which diverge when $\sigma \rightarrow 0$. Doing so, one obtains the contribution with a zero-th order as the result.

Finally, let us note the results for the renormalized vacuum stress for uniform acceleration and circular motion (Eqs. (3.6) and (4.23) in Ref. [20]). The world line of the uniformly accelerated observer is

$$z^\mu = (a^{-1} \sinh(a\tau), 0, 0, a^{-1} \cosh(a\tau)). \quad (17)$$

If one evaluates the vacuum stress at the instant that the observer is at rest, i.e., at $\tau = 0$, the result is

$$T^{\mu\nu} = \frac{11}{720\pi^2} \frac{\hbar a^4}{c^7} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (18)$$

In the laboratory frame, circular motion can be written

$$z^\mu = (\gamma\tau, R \cos(\gamma\Omega\tau), R \sin(\gamma\Omega\tau), 0), \quad (19)$$

where R is the radius of the circle, v is the velocity in the laboratory frame, $\Omega = v/R$, and $\gamma = (1 - v^2)^{-1/2}$. Here we define

$$\begin{aligned} k^\mu &= (1, 0, 0, 0), \\ \tilde{l}_1^\mu &= (0, \cos(\gamma\Omega\tau), \sin(\gamma\Omega\tau), 0), \\ \tilde{l}_2^\mu &= (0, -\sin(\gamma\Omega\tau), \cos(\gamma\Omega\tau), 0), \\ \tilde{l}_3^\mu &= (0, 0, 0, 1). \end{aligned} \quad (20)$$

Components of these vectors are defined in the laboratory frame. They are orthonormal. We can write

$$\dot{z}^\mu = \gamma k^\mu + \gamma v \tilde{l}_2^\mu. \quad (21)$$

Then, we also define

$$\begin{aligned} l_1^\mu &= \tilde{l}_1^\mu, \\ l_2^\mu &= \gamma v k^\mu + \gamma \tilde{l}_2^\mu, \\ l_3^\mu &= \tilde{l}_3^\mu. \end{aligned} \quad (22)$$

The Lorentz transformations of k^μ , \tilde{l}_1^μ , \tilde{l}_2^μ and \tilde{l}_3^μ with respect to the velocity of the observer are \dot{z}^μ , l_1^μ , l_2^μ and l_3^μ . Therefore \dot{z}^μ , l_1^μ , l_2^μ and l_3^μ are orthonormal, and they constitute the coordinate basis of the rest frame of the observer. We set the x , y and z axes along the directions of l_1^μ , l_2^μ and l_3^μ . The vacuum stress is evaluated, in the rest frame of the observer, as

$$T^{\mu\nu} = \frac{1}{1440\pi^2} \frac{\hbar\gamma^8\Omega^4 v^2}{c^5} \times \begin{bmatrix} 100 - 66\gamma^{-2} & 0 & (50 - 47\gamma^{-2})c/v & 0 \\ 0 & 30 - 22\gamma^{-2} & 0 & 0 \\ (50 - 47\gamma^{-2})c/v & 0 & 40 - 22\gamma^{-2} & 0 \\ 0 & 0 & 0 & 30 - 22\gamma^{-2} \end{bmatrix}. \quad (23)$$

(It seems that Eqs. (4.23a-c) in Ref. [20] are misprinted.) We should note that the Poynting vector is not zero, and the flux is directed along the y axis, i.e., along the Lorentz boost from the laboratory frame to the rest frame of the observer. Hacyan and Sarmiento pointed out that “if this flux is real, it should imply some friction-like effect on a rotating particle”.

2.2 The discussion of Mane

Mane suggested that the flux is related to the synchrotron radiation [10]. We outline his discussion here. We consider the case where a charged particle is moving along the orbit with radius of curvature R in the ultrarelativistic limit.

First we consider *the area in which the charged particle interacts with the electromagnetic field* (see section 14.4 of Ref. [24]). Because an ultrarelativistic particle radiates with angle $\theta \sim \gamma^{-1}$, the observer at rest at infinity observes the radiation mainly during the time that the particle rotates by $\theta \sim \gamma^{-1}$. Therefore the particle interacts with the electromagnetic field in the time $\Delta t \sim (R\theta)/v \sim R/(\gamma v)$. In that time, the radiation travels a distance $D = c\Delta t \sim (Rc)/(\gamma v)$. Therefore the electromagnetic wave radiated in Δt spreads out in the area $A \sim (\theta D)^2$ on a surface orthogonal to the orbit. This area is invariant for the Lorentz transformation from the laboratory frame to the rest frame, because the surface is orthogonal to the orbit. We regard A as *the area in which the charged particle interacts with the electromagnetic field*.

We can write the Poynting flux in Eq. (23) in the form

$$pl_2^\mu = \frac{1}{1440\pi^2} \frac{\hbar\gamma^8\Omega^4 v}{c^4} (50 - 47\gamma^{-2}) l_2^\mu, \quad (24)$$

where l_2^μ is given as

$$l_2^\mu = ((\gamma v)/c, -\gamma \sin(\gamma\Omega\tau), \gamma \cos(\gamma\Omega\tau), 0). \quad (25)$$

Since p is proportional to \hbar , the flux becomes zero in the classical limit. But if the particle couples with the flux by the fine structure constant $\alpha = e^2/\hbar c$, the contribution of \hbar vanishes, and the effect on the particle becomes classical. The recoil induced by the flux of the vacuum fluctuation on the four-momentum of the particle per unit proper time is

$$\alpha A p l_2^\mu \sim \frac{e^2 \gamma^4 \Omega^2 v}{c^3} l_2^\mu. \quad (26)$$

In the laboratory frame, the energy loss of the particle per unit laboratory time is given by the Larmor formula

$$I = \frac{2e^2}{3c^3} (\gamma^2 \Omega v)^2. \quad (27)$$

This is related to the damping force \mathbf{F} in the form $I = \mathbf{F} \cdot \mathbf{v}$ and therefore the recoil induced by synchrotron radiation on the four-momentum of the particle per unit proper time is

$$\gamma(I/c, \mathbf{F}) \sim \frac{e^2 \gamma^4 \Omega^2 v}{c^3} l_2^\mu. \quad (28)$$

Hence, we find that, if one assume that the charged particle interacts with the vacuum flux by the coupling α , the order of recoil of the particle induced by this interaction is equal to that derived by the Larmor formula in the ultrarelativistic limit.

3 Expansion of a retarded field

To investigate the relation between the vacuum stress and the bremsstrahlung, we evaluate the stress-energy tensor of a self-field generated by a charged particle, at the position of the particle. We use a point charge and adopt the retarded field as the self-field. Therefore, the self-stress is now divergent if we evaluate it as it is. Hence we must remove the divergent contributions with some procedure. First, we construct the expansion of the retarded field in powers of the distance from the point charge. By doing this, we calculate the terms of zero-th order in the expansion of the stress-energy tensor, and we regard the result as the renormalized stress-energy tensor. This procedure reminds us of our interpretation of the renormalization of the vacuum stress, where we expanded the stress-energy tensor in powers of the proper time.

In calculating the expansion of the field, we adopt the method used by Dirac in Ref. [23], which is followed in this section and in Appendix A. (Dirac used the expansion for calculating the energy-momentum flow out of the world tube surrounding the world line of a point charge. However, our aim is not to investigate this quantity. Our aim is to evaluate the each component of $T_{\mu\nu}$.)

The retarded potential generated by a 4-current $j^\mu(x)$ is given in the form

$$A_{\text{ret}}^\mu(x) = 4\pi \int d^4x' D_r(x - x') j^\mu(x'), \quad (29)$$

where

$$D_r(x - x') = \frac{1}{2\pi} \theta(x_0 - x'_0) \delta[(x - x')^2] \quad (30)$$

is the retarded Green function. We define the world line of the point charge as $z^\mu(\tau)$, where τ is the proper time. Then the 4-current of the point charge with charge e is

$$j^\mu(x) = e \int_{-\infty}^{\infty} d\tau \dot{z}^\mu(\tau) \delta^{(4)}[x - z(\tau)]. \quad (31)$$

The dot above z represents differentiation with respect to the proper time. Substituting Eqs. (30) and (31) into Eq. (29), we have

$$\begin{aligned} A_{\text{ret}}^\mu(x) &= 2e \int_{-\infty}^{\infty} d\tau \dot{z}^\mu \theta(x_0 - z_0) \delta[(x - z)^2] \\ &= 2e \int d\tau \dot{z}^\mu \delta[(x - z)^2], \end{aligned} \quad (32)$$

where, in the last, integration is taken from $-\infty$ to some value of τ intermediate between the retarded and advanced times. We now have

$$\begin{aligned} \partial_\nu A_{\mu,\text{ret}}(x) &= 4e \int d\tau \dot{z}_\mu(x_\nu - z_\nu) \delta'[(x - z)^2] \\ &= -2e \int d\tau \frac{\dot{z}_\mu(x_\nu - z_\nu)}{\dot{z} \cdot (x - z)} \frac{d}{d\tau} \delta[(x - z)^2] \\ &= 2e \int d\tau \frac{d}{d\tau} \left[\frac{\dot{z}_\mu(x_\nu - z_\nu)}{\dot{z} \cdot (x - z)} \right] \delta[(x - z)^2]. \end{aligned} \quad (33)$$

Thus the retarded field of the point charge becomes

$$\begin{aligned} F_{\mu\nu,\text{ret}}(x) &= \partial_\mu A_{\nu,\text{ret}}(x) - \partial_\nu A_{\mu,\text{ret}}(x) \\ &= -2e \int d\tau \frac{d}{d\tau} \left[\frac{\dot{z}_\mu(x_\nu - z_\nu) - \dot{z}_\nu(x_\mu - z_\mu)}{\dot{z} \cdot (x - z)} \right] \delta[(x - z)^2] \\ &= \frac{e}{\dot{z} \cdot (z - x)} \frac{d}{d\tau} \frac{\dot{z}_\mu(z_\nu - x_\nu) - \dot{z}_\nu(z_\mu - x_\mu)}{\dot{z} \cdot (z - x)}, \end{aligned} \quad (34)$$

where z^μ is evaluated at the retarded time in the last equation.

Here we set

$$x^\mu = z^\mu(\tau_0) + \gamma^\mu \quad (35)$$

and expand Eq. (34) in powers of γ^μ . At that time, we choose τ_0 to satisfy

$$\dot{z}(\tau_0) \cdot \gamma = 0. \quad (36)$$

If one choose the frame in which the charge is instantaneously at rest at the instant $\tau = \tau_0$, γ^μ has only spatial components, so that $x^0 = z(\tau_0)^0$. Also, $\epsilon \equiv \sqrt{-\gamma \cdot \gamma}$ is the distance from $z(\tau_0)$ to \mathbf{x} in this frame. Therefore, the expansion with respect to ϵ is equivalent to that with respect to the distance from the point charge in the instantaneous rest frame of the point charge. We point out that Dirac calculated the expansion of $[1 - \gamma \cdot \ddot{z}]^{1/2} F_{\mu\nu,\text{ret}}$ to obtain the energy-momentum flow out of the world tube, but we expand $F_{\mu\nu,\text{ret}}$, because our purpose is different from that of Dirac. The details of the culculation are complicated, and therefore they are given in Appendix A. Before we give the result of the calculation, some notation is defined:

$$\begin{aligned} n^\mu &= \epsilon^{-1} \gamma^\mu, \\ (m)^\mu &= \frac{d^m z^\mu}{d\tau^m}, \\ \Delta_m &= n \cdot (m), \\ \alpha_2 &= (2) \cdot (2), \\ \alpha_3 &= (3) \cdot (3). \end{aligned} \quad (37)$$

The expansion of $F_{\mu\nu,\text{ret}}$ in powers of ϵ is derived as follows:

$$\begin{aligned} F_{\mu\nu,\text{ret}}(x) &= e f_{\mu\nu}^{(-2)} \epsilon^{-2} + e f_{\mu\nu}^{(-1)} \epsilon^{-1} + e f_{\mu\nu}^{(0)} + e f_{\mu\nu}^{(1)} \epsilon + e f_{\mu\nu}^{(2)} \epsilon^2 - (\mu \leftrightarrow \nu) + O(\epsilon^3), \\ f_{\mu\nu}^{(-2)} &= n_\mu(1)_\nu, \\ f_{\mu\nu}^{(-1)} &= \frac{1}{2} \Delta_2 n_\mu(1)_\nu - \frac{1}{2} (2)_\mu(1)_\nu, \\ f_{\mu\nu}^{(0)} &= \left[\frac{3}{8} (\Delta_2)^2 - \frac{1}{8} \alpha_2 \right] n_\mu(1)_\nu - \frac{3}{4} \Delta_2 (2)_\mu(1)_\nu - \frac{1}{2} n_\mu(3)_\nu + \frac{2}{3} (3)_\mu(1)_\nu, \\ f_{\mu\nu}^{(1)} &= \left[\frac{5}{16} (\Delta_2)^3 - \frac{5}{16} \Delta_2 \alpha_2 - \frac{1}{8} \Delta_4 + \frac{1}{6} \dot{\alpha}_2 \right] n_\mu(1)_\nu + \left[-\frac{1}{2} \Delta_3 + \frac{1}{3} \alpha_2 \right] n_\mu(2)_\nu \\ &\quad + \left[-\frac{15}{16} (\Delta_2)^2 + \frac{2}{3} \Delta_3 - \frac{5}{16} \alpha_2 \right] (2)_\mu(1)_\nu - \frac{3}{4} \Delta_2 n_\mu(3)_\nu \\ &\quad + \frac{4}{3} \Delta_2 (3)_\mu(1)_\nu + \frac{1}{3} n_\mu(4)_\nu - \frac{3}{8} (4)_\mu(1)_\nu - \frac{1}{4} (3)_\mu(2)_\nu, \\ f_{\mu\nu}^{(2)} &= \left[\frac{35}{128} (\Delta_2)^4 - \frac{35}{64} (\Delta_2)^2 \alpha_2 - \frac{5}{16} \Delta_2 \Delta_4 + \frac{1}{2} \Delta_2 \dot{\alpha}_2 - \frac{5}{24} (\Delta_3)^2 - \frac{35}{384} (\alpha_2)^2 \right. \\ &\quad \left. + \frac{1}{3} \Delta_3 \alpha_2 + \frac{1}{15} \Delta_5 - \frac{3}{32} \ddot{\alpha}_2 + \frac{1}{48} \alpha_3 \right] n_\mu(1)_\nu \\ &\quad + \left[-\frac{5}{4} \Delta_2 \Delta_3 + \Delta_2 \alpha_2 + \frac{1}{3} \Delta_4 - \frac{5}{16} \dot{\alpha}_2 \right] n_\mu(2)_\nu \\ &\quad + \left[-\frac{35}{32} (\Delta_2)^3 + 2 \Delta_2 \Delta_3 + \frac{1}{4} \dot{\alpha}_2 - \frac{5}{16} \Delta_4 - \frac{35}{32} \Delta_2 \alpha_2 \right] (2)_\mu(1)_\nu \end{aligned}$$

$$\begin{aligned}
& + \left[-\frac{15}{16}(\Delta_2)^2 + \frac{2}{3}\Delta_3 - \frac{5}{16}\alpha_2 \right] n_\mu(3)_\nu + \left[2(\Delta_2)^2 - \frac{5}{6}\Delta_3 + \frac{1}{3}\alpha_2 \right] (3)_\mu(1)_\nu \\
& + \frac{2}{3}\Delta_2 n_\mu(4)_\nu - \frac{15}{16}\Delta_2(4)_\mu(1)_\nu - \frac{5}{8}\Delta_2(3)_\mu(2)_\nu \\
& - \frac{1}{8}n_\mu(5)_\nu + \frac{2}{15}(5)_\mu(1)_\nu + \frac{1}{6}(4)_\mu(2)_\nu,
\end{aligned} \tag{38}$$

where the functions of τ in the expansion are evaluated at the time $\tau = \tau_0$.

Here we would like to point out the work of Teitelboim [22], who showed that the radiation reaction force of the Lorentz-Dirac equation can be derived by simply averaging the angular dependence of above expansion of the field. We summarize his discussion in the following.

He proposed to evaluate the value of the retarded field at the particle's own position and the force acting on the particle. However, obviously there are two problems. The first is that the retarded field diverges at the position of the particle. The second is that the "limit" of the retarded field depends on the direction along which the singularity is approached. In fact, in Eq. (38), the angular dependence, n^μ , is included in the coefficients of the expansion.

He avoided the second problem by simply averaging the angular dependence of Eq. (38) in the instantaneous rest frame of the charge. In this frame, one can write $n^\mu = (0, n_x, n_y, n_z)$, so that $\mathbf{n} = (n_x, n_y, n_z)$ is the unit vector directed from the position of the charge to reference point of the field. He averaged Eq. (38) to the order of $O(\epsilon^0)$. The terms in Eq. (38) which contain odd n^μ 's vanish when the average is performed, because the signs of these terms change when the direction of \mathbf{n} is reversed. Only remained term is $\Delta_2 n_\mu(1)_\nu$ in $f_{\mu\nu}^{(-2)}$. If one expresses an angular averaged function by drawing a bar over the quantity, it follows that

$$\overline{\Delta_2 \mathbf{n}} = -\overline{(\mathbf{n} \cdot \ddot{\mathbf{z}}) \mathbf{n}} = -\overline{\cos^2 \theta} \ddot{\mathbf{z}} = -\frac{1}{3} \ddot{\mathbf{z}}, \tag{39}$$

where θ is the angle between $\ddot{\mathbf{z}}$ and \mathbf{n} . This relation is easily rewritten in the covariant form

$$\overline{\Delta_2 n^\mu} = -\frac{1}{3}(2)^\mu. \tag{40}$$

Then it follows that

$$\overline{F_{\mu\nu,\text{ret}}} = -\frac{2e}{3}(2)_\mu(1)_\nu \epsilon^{-1} + \frac{2e}{3}(3)_\mu(1)_\nu - (\mu \leftrightarrow \nu) + O(\epsilon). \tag{41}$$

Thus the Lorentz force acting on the charge is obtained as

$$e \overline{F_{\text{ret}}^{\mu\nu}}(1)_\nu = - \left(\lim_{\epsilon \rightarrow 0} \frac{2e^2}{3\epsilon} (2)^\mu \right) + \frac{2e^2}{3} \{(3)^\mu + \alpha_2(1)^\mu\}. \tag{42}$$

The first term on the right-hand side, which diverges in the limit $\epsilon \rightarrow 0$, is interpreted as the infinite Coulomb mass of the point charge, and this is absorbed in the usual

way into the observed finite mass of the particle. The second term represents the radiation reaction force, which is equivalent to the radiation reaction force of the Lorentz-Dirac equation [22].

4 Evaluation of self-stress

In this section we calculate the terms with zero-th order in the expansion of the stress-energy tensor for a point charge with uniform acceleration and circular motion. We evaluate them in the rest frame of the charge at the instant $\tau = \tau_0$.

First let us calculate for the case of a point charge with uniform acceleration a . The world line of the charge is expressed by Eq. (17). The charge is at rest at $\tau = 0$, so we evaluate it at $\tau = 0$. Inserting

$$\begin{aligned} (2)^\mu &= (0, 0, 0, a) \equiv am^\mu, \\ (3)^\mu &= a^2(1)^\mu, \\ (4)^\mu &= a^3m^\mu, \\ (5)^\mu &= a^4(1)^\mu \end{aligned} \quad (43)$$

into Eq. (38), we have

$$\begin{aligned} F_{\mu\nu, \text{ret}} &= ef_{\mu\nu, \text{uni}}^{(-2)}\epsilon^{-2} + ef_{\mu\nu, \text{uni}}^{(-1)}a\epsilon^{-1} + ef_{\mu\nu, \text{uni}}^{(0)}a^2 + ef_{\mu\nu, \text{uni}}^{(1)}a^3\epsilon + ef_{\mu\nu, \text{uni}}^{(2)}a^4\epsilon^2 \\ &\quad - (\mu \leftrightarrow \nu) + O(\epsilon^3), \\ f_{\mu\nu, \text{uni}}^{(-2)} &= n_\mu(1)_\nu, \\ f_{\mu\nu, \text{uni}}^{(-1)} &= \frac{1}{2}(n \cdot m)n_\mu(1)_\nu - \frac{1}{2}m_\mu(1)_\nu, \\ f_{\mu\nu, \text{uni}}^{(0)} &= \frac{3}{8}\left[-1 + (n \cdot m)^2\right]n_\mu(1)_\nu - \frac{3}{4}(n \cdot m)m_\mu(1)_\nu, \\ f_{\mu\nu, \text{uni}}^{(1)} &= \left[\frac{5}{16}(n \cdot m)^3 - \frac{9}{16}(n \cdot m)\right]n_\mu(1)_\nu + \left[-\frac{15}{16}(n \cdot m)^2 + \frac{3}{16}\right]m_\mu(1)_\nu, \\ f_{\mu\nu, \text{uni}}^{(2)} &= \left[\frac{15}{128} - \frac{45}{64}(n \cdot m)^2 + \frac{35}{128}(n \cdot m)^4\right]n_\mu(1)_\nu \\ &\quad + \left[\frac{15}{32}(n \cdot m) - \frac{35}{32}(n \cdot m)^3\right]m_\mu(1)_\nu. \end{aligned} \quad (44)$$

Here we note that the terms with $n_\mu m_\nu - n_\nu m_\mu$ have vanished. By using

$$\begin{aligned} [n_\mu(1)_\alpha - (1)_\mu n_\alpha][n^\alpha(1)_\nu - (1)^\alpha n_\nu] &= -n_\mu n_\nu + (1)_\mu(1)_\nu, \\ [m_\mu(1)_\alpha - (1)_\mu m_\alpha][m^\alpha(1)_\nu - (1)^\alpha m_\nu] &= -m_\mu m_\nu + (1)_\mu(1)_\nu, \\ [n_\mu(1)_\alpha - (1)_\mu n_\alpha][m^\alpha(1)_\nu - (1)^\alpha m_\nu] &= -n_\mu m_\nu - (n \cdot m)(1)_\mu(1)_\nu, \end{aligned} \quad (45)$$

the zero-th terms in $e^{-2}a^{-4}F_{\alpha, \text{ret}}^\mu F_{\text{ret}}^{\alpha\nu}$ are obtained as

$$\left[-\frac{3}{8} + \frac{9}{4}(n \cdot m)^2 - (n \cdot m)^4\right]n^\mu n^\nu + \left[\frac{3}{16} - 3(n \cdot m)^2 + 5(n \cdot m)^4\right](1)^\mu(1)^\nu$$

$$+ \left[-\frac{9}{8}(n \cdot m) + 2(n \cdot m)^3 \right] (n^\mu m^\nu + m^\mu n^\nu) + \left[\frac{3}{16} - \frac{3}{2}(n \cdot m)^2 \right] m^\mu m^\nu, \quad (46)$$

and the zero-th terms in $e^{-2}a^{-4}F_{\mu\nu,\text{ret}}F_{\text{ret}}^{\nu\mu}$ are obtained as

$$\frac{3}{8} - 6(n \cdot m)^2 + 10(n \cdot m)^4. \quad (47)$$

Substituting these into the expression of the stress-energy tensor

$$T^{\mu\nu} = \frac{1}{4\pi} \left(F^{\mu\alpha} F_\alpha^\nu - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\beta\alpha} \right), \quad (48)$$

we obtain the zero-th terms in the expansion of $T^{\mu\nu}$ in powers of ϵ . From Eq. (46), it is found that the zero-th terms of the Poynting vector T^{0i} , where the Roman index i represents a spatial component, are zero. This result is explained by the well-known fact that, in the rest frame of a point charge with uniform acceleration, the Poynting vector of the retarded field vanishes because of the null magnetic field [16].

We wish to evaluate the zero-th terms in the expansion at the position of the charge, but Eqs. (46), and (47) are indefinite at that position because of the angular dependence n^μ . So let us proceed in the same manner as in the method used in the previous section. That is, let us consider the expansion, in powers of ϵ , of *the angular average of $T^{\mu\nu}$ around the point charge*, and calculate the zero-th terms of the expansion. The average is taken in the rest frame of the charge. Using the equations

$$\overline{n_z^2} = \frac{1}{3}, \quad \overline{n_z^4} = \frac{1}{5}, \quad \overline{n_z^6} = \frac{1}{7}, \quad \overline{n_x^2 n_z^2} = \frac{1}{15}, \quad \overline{n_x^2 n_z^4} = \frac{1}{35}. \quad (49)$$

for evaluating Eqs. (46) and (47), and noting $(n \cdot m) = -n_z$, we have

$$e^{-2}a^{-4}\overline{(F_{\alpha,\text{ret}}^\mu F_{\text{ret}}^{\alpha\nu})_0} = \frac{1}{560} \begin{bmatrix} 105 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -101 \end{bmatrix}, \quad (50)$$

and

$$e^{-2}a^{-4}\overline{(F_{\mu\nu,\text{ret}}F_{\text{ret}}^{\nu\mu})_0} = \frac{3}{8}. \quad (51)$$

Inserting these into Eq. (48), we obtain

$$\begin{aligned} \overline{T^{\mu\nu}} &= \sum_{i=-4}^{\infty} \overline{(T^{\mu\nu})_i} \epsilon^i, \\ \overline{(T^{\mu\nu})_0} &= \pi\alpha \cdot \frac{1}{4480\pi^2} \frac{\hbar a^4}{c^7} \begin{bmatrix} 105 & 0 & 0 & 0 \\ 0 & 101 & 0 & 0 \\ 0 & 0 & 101 & 0 \\ 0 & 0 & 0 & -97 \end{bmatrix}. \end{aligned} \quad (52)$$

Next, let us calculate for a point charge with circular motion. The world line of the charge is given by Eq. (19). We have

$$\begin{aligned}(2)^\mu &= -\gamma^2 \Omega v l_1^\mu, \\ (3)^\mu &= -\gamma^4 \Omega^2 v (l_2^\mu - v(1)^\mu), \\ (4)^\mu &= \gamma^4 \Omega^3 v l_1^\mu, \\ (5)^\mu &= \gamma^6 \Omega^4 v (l_2^\mu - v(1)^\mu).\end{aligned}\quad (53)$$

Inserting these into Eq. (38), and noting $(n \cdot l_1) = -n_x$, $(n \cdot l_2) = -n_y$, we find

$$\begin{aligned}F_{\mu\nu,\text{ret}} &= e f_{\mu\nu,\text{cir}}^{(-2)} \epsilon^{-2} + e f_{\mu\nu,\text{cir}}^{(-1)} \gamma^2 \Omega \epsilon^{-1} + e f_{\mu\nu,\text{cir}}^{(0)} \gamma^4 \Omega^2 + e f_{\mu\nu,\text{cir}}^{(1)} \gamma^6 \Omega^3 \epsilon + e f_{\mu\nu,\text{cir}}^{(2)} \gamma^8 \Omega^4 \epsilon^2 \\ &\quad - (\mu \leftrightarrow \nu) + O(\epsilon^3), \\ f_{\mu\nu,\text{cir}}^{(-2)} &= n_\mu(1)_\nu, \\ f_{\mu\nu,\text{cir}}^{(-1)} &= \frac{1}{2} v n_x n_\mu(1)_\nu + \frac{1}{2} v l_{1\mu}(1)_\nu, \\ f_{\mu\nu,\text{cir}}^{(0)} &= \left[\frac{3}{8} v^2 n_x^2 - \frac{3}{8} v^2 \right] n_\mu(1)_\nu + \frac{3}{4} v^2 n_x l_{1\mu}(1)_\nu + \frac{1}{2} v n_\mu l_{2\nu} - \frac{2}{3} v l_{2\mu}(1)_\nu, \\ f_{\mu\nu,\text{cir}}^{(1)} &= \left[\frac{5}{16} v^3 n_x^3 - \frac{9}{16} v^3 n_x + \frac{1}{8} v n_x \right] n_\mu(1)_\nu + \left[\frac{1}{2} v^2 n_y + \frac{1}{3} v \right] n_\mu l_{1\nu} \\ &\quad + \left[\frac{15}{16} v^3 n_x^2 - \frac{2}{3} v^2 n_y - \frac{3}{16} v^3 - \frac{3}{8} v \right] l_{1\mu}(1)_\nu \\ &\quad + \frac{3}{4} v^2 n_x n_\mu l_{2\nu} - \frac{4}{3} v^2 n_x l_{2\mu}(1)_\nu + \frac{1}{4} v^2 l_{1\mu} l_{2\nu}, \\ f_{\mu\nu,\text{cir}}^{(2)} &= \left[\frac{35}{128} v^4 n_x^4 - \frac{45}{64} v^4 n_x^2 + \frac{5}{16} v^2 n_x^2 - \frac{5}{24} v^2 n_y^2 + \frac{15}{128} v^4 + \frac{2}{5} v^3 n_y - \frac{1}{15} v n_y \right. \\ &\quad \left. + \frac{5}{48} v^2 \right] n_\mu(1)_\nu + \left[\frac{5}{4} v^3 n_x n_y + v^2 n_x \right] n_\mu l_{1\nu} \\ &\quad + \left[\frac{35}{32} v^4 n_x^3 - 2v^3 n_x n_y - \frac{5}{4} v^2 n_x - \frac{15}{32} v^4 n_x \right] l_{1\mu}(1)_\nu \\ &\quad + \left[\frac{15}{16} v^3 n_x^2 - \frac{2}{3} v^2 n_y - \frac{3}{16} v^3 - \frac{1}{8} v \right] n_\mu l_{2\nu} \\ &\quad + \left[-2v^3 n_x^2 + \frac{5}{6} v^2 n_y + \frac{1}{5} v^3 + \frac{2}{15} v \right] l_{2\mu}(1)_\nu + \frac{5}{8} v^3 n_x l_{1\mu} l_{2\nu}.\end{aligned}\quad (54)$$

The terms of zero-th order in $e^{-2} \gamma^{-8} \Omega^{-4} F^{\mu\alpha} F_\alpha^\nu$ are

$$\begin{aligned}&\left[-v^4 n_x^4 + \frac{9}{4} v^4 n_x^2 - \frac{3}{4} v^2 n_x^2 + \frac{5}{12} v^2 n_y^2 - \frac{3}{8} v^4 - \frac{4}{5} v^3 n_y + \frac{2}{15} v n_y + \frac{1}{24} v^2 \right] n^\mu n^\nu \\ &+ \left[5v^4 n_x^4 - 3v^4 n_x^2 - 2v^2 n_x^2 + \frac{5}{4} v^2 n_y^2 + \frac{3}{16} v^4 + \frac{31}{30} v^3 n_y + \frac{2}{15} v n_y + \frac{5}{18} v^2 \right. \\ &\quad \left. - \frac{21}{2} v^3 n_x^2 n_y \right] (1)^\mu(1)^\nu + \left[v^3 n_x n_y + \frac{4}{3} v^2 n_x \right] ((1)^\mu l_1^\nu + l_1^\mu(1)^\nu) \\ &+ \left[-3v^3 n_x^2 n_y - \frac{7}{6} v^2 n_x^2 + \frac{2}{3} v^2 n_y^2 + \frac{1}{8} v^3 n_y + \frac{1}{8} v n_y + \frac{1}{6} v^2 \right] (n^\mu(1)^\nu + (1)^\mu n^\nu)\end{aligned}$$

$$\begin{aligned}
& + \left[-2v^4 n_x^3 + \frac{7}{3} v^3 n_x n_y + \frac{11}{8} v^2 n_x + \frac{9}{8} v^4 n_x \right] (n^\mu l_1^\nu + l_1^\mu n^\nu) \\
& + \left[3v^3 n_x^2 - v^2 n_y - \frac{1}{4} v^3 - \frac{1}{8} v \right] (l_2^\mu (1)^\nu + (1)^\mu l_2^\nu) \\
& + \left[\frac{35}{12} v^3 n_x^2 - \frac{13}{12} v^2 n_y - \frac{9}{20} v^3 - \frac{2}{15} v \right] (n^\mu l_2^\nu + l_2^\mu n^\nu) \\
& + \left[-\frac{3}{2} v^4 n_x^2 + \frac{2}{3} v^3 n_y + \frac{3}{16} v^4 + \frac{3}{8} v^2 \right] l_1^\mu l_1^\nu + \frac{7}{6} v^3 n_x (l_1^\mu l_2^\nu + l_2^\mu l_1^\nu) - \frac{7}{36} v^2 l_2^\mu l_2^\nu,
\end{aligned} \tag{55}$$

and the terms of zero-th order in $e^{-2}\gamma^{-8}\Omega^{-4}F_{\mu\nu}F^{\nu\mu}$ are

$$10v^4 n_x^4 - 6v^4 n_x^2 - 4v^2 n_x^2 + 3v^2 n_y^2 + \frac{3}{8} v^4 + \frac{31}{15} v^3 n_y + \frac{4}{15} v n_y + \frac{1}{18} v^2 - 21v^3 n_x^2 n_y. \tag{56}$$

Their angular averages are

$$\begin{aligned}
\frac{1}{e^2 \gamma^8 \Omega^4} \overline{(F^{\mu\alpha} F_\alpha^\nu)_0} &= \frac{v^2}{5040} \times \\
\begin{bmatrix} 1085 - 945\gamma^{-2} & 0 & (2562 - 2982\gamma^{-2})v^{-1} & 0 \\ 0 & 5055 + 909\gamma^{-2} & 0 & 0 \\ (2562 - 2982\gamma^{-2})v^{-1} & 0 & -4400 + 18\gamma^{-2} & 0 \\ 0 & 0 & 0 & -60 + 18\gamma^{-2} \end{bmatrix},
\end{aligned} \tag{57}$$

and

$$e^{-2}\gamma^{-8}\Omega^{-4}\overline{(F_{\mu\nu}F^{\nu\mu})_0} = \frac{v^2}{72}[7 - 27\gamma^{-2}]. \tag{58}$$

Substituting these into Eq. (48), we find that the zero-th term in the expansion of the angular averaged stress-energy tensor is

$$\begin{aligned}
\overline{(T^{\mu\nu})_0} &= \pi\alpha \cdot \frac{1}{40320\pi^2} \frac{\hbar\gamma^8\Omega^4 v^2}{c^5} \times \\
\begin{bmatrix} 1925 - 945\gamma^{-2} & 0 & (5124 - 5964\gamma^{-2})c/v & 0 \\ 0 & 10355 + 873\gamma^{-2} & 0 & 0 \\ (5124 - 5964\gamma^{-2})c/v & 0 & -8555 - 909\gamma^{-2} & 0 \\ 0 & 0 & 0 & 125 - 909\gamma^{-2} \end{bmatrix}.
\end{aligned} \tag{59}$$

5 Discussion

Before comparing the self-stress with the vacuum stress, we should comment on the physical relevance of the self-stress which we have calculated. It should be noted

that, although zero-th terms in the expansion of the self-stress themselves would be mathematically well defined quantities, our evaluation includes an artificial procedure in which we average the angular dependence of the quantities. (Although Teitelboim was able to rederive the radiation reaction force of the Lorentz-Dirac equation by applying this angular average, there is no guarantee that the method of angular average is valid even in the evaluation of the self-stress.) Moreover, even if we can obtain a natural definition of the self-stress at the position of the particle, it is not clear whether we can give this quantity the definite meaning when the physical predictability is considered. Our interest in the calculation is limited whether we can find out the trace of the Unruh-like effect in the calculation involving the self-field.

Let us compare the vacuum stress, Eq. (18) or Eq. (23), with the self-stress, Eq. (52) or Eq. (59). In uniform acceleration, both of them are proportional to the 4th power of the acceleration a . In the case of circular motion, both of them are proportional to the 4th power of $\gamma^2\Omega$, and the degrees of v are equal. Therefore, roughly speaking, the order of vacuum stress multiplied by $\pi\alpha$ is equivalent to that of the self-stress. Let us now consider the situation in more detail. In uniform acceleration, while the vacuum stress represents an isotropic thermal bath of photons, as for the self-stress, the magnitude of the radiation pressure is close to the energy density. Moreover, the radiation pressure is anisotropic, and the tension acts along the direction of the acceleration. In circular motion, the self-stress represents tension along the y axis. (The signs of the components of the stress-energy tensor are often changed if we choose another method of evaluation. See the following paragraph and Appendix B.) Furthermore, renormalization of $F_{\mu\nu}F^{\nu\mu}$ for both uniformly accelerated and circular motion (Eqs. (51) and (58)) gives nonzero values, in contrast to the fact that the vacuum expectation value of $F_{\mu\nu}F^{\nu\mu}$ (Eq. (7)) is zero for arbitrary motion of the observer. Therefore, the resemblance between the two stress-energy tensors is not perfect. However, it would be rather impressive that the degrees of a , $\gamma^2\Omega$ and v , are *all* equal. One cannot discard the possibility that the self-stress which we have calculated reflects some indication of the Unruh-like effect.

We note that, in the derivation of the self-stress, we have used an expansion in powers of the distance from the charge. However, alternatively, we could construct the expansion in powers of the retarded time by substituting $z(\tau_0)$ for x in Eq. (34). If this is done, expansion coefficients do not include the directional unit vector n^μ , and thus the angular dependence disappears. We discuss this alternative method in Appendix B. In this method, we find, similarly, that the order of the vacuum stress multiplied by $\pi\alpha$ is equivalent to that of the self-stress. Furthermore, we can consider the spectrum of the self-stress in this case, because the Fourier transform with respect to time can be taken, and in fact we do so in the Appendix. But our trial calculation for the uniform acceleration does not lead to a clear spectrum of the thermal bath, and it results in a rather awkward form.

As stated above, the physical relevance of the self-stress which we have evaluated is not clear. If one wish to investigate the more detailed relation between bremsstrahlung

and the Unruh-like effect, our approach would not be so effective, in spite of a very long calculation. But the resemblance between the self-stress and the vacuum stress we have revealed might offer a hint to investigate this subject.

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Appendix A

—*Expansion of a Field in Powers of ϵ* —

In this Appendix, we perform the derivation of Eq. (38). For simplicity, we set $\tau_0 = 0$. First, we consider a Taylor expansion of the right-hand side of Eq. (34) with respect to the proper time, around $\tau = \tau_0 = 0$. Next, we translate this expansion into an expansion in powers of ϵ . In the following, the coefficients of the expansion are evaluated at $\tau = 0$, and the time is omitted.

A.1. Taylor expansion of the field

Let us expand $z(\tau) - x$ and $\dot{z}(\tau)$ around $\tau = 0$. Because the retarded time τ depends on ϵ in the form $\tau \sim -\epsilon$, we can write, while keeping the order of ϵ in mind,

$$z(\tau) - x = -\gamma + (1)\tau + \frac{1}{2}(2)\tau^2 + \frac{1}{6}(3)\tau^3 + \frac{1}{24}(4)\tau^4 + \frac{1}{120}(5)\tau^5 + O(\epsilon^6), \quad (60)$$

$$\dot{z}(\tau) = (1) + (2)\tau + \frac{1}{2}(3)\tau^2 + \frac{1}{6}(4)\tau^3 + \frac{1}{24}(5)\tau^4 + O(\epsilon^5). \quad (61)$$

Using these equations, we find

$$\begin{aligned} \dot{z}(\tau) \cdot [z(\tau) - x] &= [1 - \Delta_2\epsilon]\tau - \frac{1}{2}\Delta_3\epsilon\tau^2 - \frac{1}{6}[\alpha_2 + \Delta_4\epsilon]\tau^3 - \frac{1}{48}[5\dot{\alpha}_2 + 2\Delta_5\epsilon]\tau^4 \\ &\quad + \frac{1}{240}[-9\ddot{\alpha}_2 + 2\alpha_3]\tau^5 + O(\epsilon^6), \end{aligned} \quad (62)$$

where we use the following equations:

$$\begin{aligned} (1) \cdot (1) &= 1, \\ (1) \cdot (2) &= 0, \end{aligned}$$

$$\begin{aligned}
(1) \cdot (3) &= -\alpha_2, \\
(2) \cdot (3) &= \frac{1}{2}\dot{\alpha}_2, \\
(1) \cdot (4) &= -\frac{3}{2}\dot{\alpha}_2, \\
(2) \cdot (4) &= \frac{1}{2}\ddot{\alpha}_2 - \alpha_3, \\
(1) \cdot (5) &= -2\ddot{\alpha}_2 + \alpha_3.
\end{aligned} \tag{63}$$

We should note that, because we keep the order of ϵ in mind, the coefficient with τ^5 in Eq. (62) is not equivalent to the fifth order term in the Taylor expansion with respect to τ . By using the above expression, the expansion of the reciprocal of Eq. (62), keeping the order of ϵ in mind, is obtained as

$$\begin{aligned}
\frac{\tau}{\dot{z}(\tau) \cdot [z(\tau) - x]} &= f_{(0)} + f_{(1)}\tau + \frac{1}{2}f_{(2)}\tau^2 + \frac{1}{6}f_{(3)}\tau^3 + \frac{1}{24}f_{(4)}\tau^4 + O(\epsilon^5), \\
f_{(0)} &= 1 + \Delta_2\epsilon + (\Delta_2)^2\epsilon^2 + (\Delta_2)^3\epsilon^3 + (\Delta_2)^4\epsilon^4, \\
f_{(1)} &= \frac{1}{2}\Delta_3\epsilon + \Delta_2\Delta_3\epsilon^2 + \frac{3}{2}(\Delta_2)^2\Delta_3\epsilon^3, \\
f_{(2)} &= \frac{1}{3}\alpha_2 + \left[\frac{1}{3}\Delta_4 + \frac{2}{3}\Delta_2\alpha_2 \right] \epsilon + \left[(\Delta_2)^2\alpha_2 + \frac{2}{3}\Delta_2\Delta_4 + \frac{1}{2}(\Delta_3)^2 \right] \epsilon^2, \\
f_{(3)} &= \frac{5}{8}\dot{\alpha}_2 + \left[\Delta_3\alpha_2 + \frac{1}{4}\Delta_5 + \frac{5}{4}\Delta_2\dot{\alpha}_2 \right] \epsilon, \\
f_{(4)} &= \frac{2}{3}(\alpha_2)^2 + \frac{9}{10}\ddot{\alpha}_2 - \frac{1}{5}\alpha_3.
\end{aligned} \tag{64}$$

By substituting Eqs. (60), (61) and (64) into the part of Eq. (34) where $d/d\tau$ acts, we fix the part in powers of τ up to the order $O(\epsilon^4)$. After this, we carry out the differentiation.

Now, we can construct the expansion of Eq. (34) in powers of ϵ by using the relation

$$\begin{aligned}
\tau &= -\epsilon - g_{(1)}\epsilon^2 - \frac{1}{2}g_{(2)}\epsilon^3 - \frac{1}{6}g_{(3)}\epsilon^4 - \frac{1}{24}g_{(4)}\epsilon^5 + O(\epsilon^6), \\
g_{(1)} &= \frac{1}{2}\Delta_2, \\
g_{(2)} &= \frac{3}{4}(\Delta_2)^2 - \frac{1}{3}\Delta_3 + \frac{1}{12}\alpha_2, \\
g_{(3)} &= \frac{15}{8}(\Delta_2)^3 - 2\Delta_2\Delta_3 + \frac{5}{8}\Delta_2\alpha_2 + \frac{1}{4}\Delta_4 - \frac{1}{8}\dot{\alpha}_2, \\
g_{(4)} &= \frac{105}{16}(\Delta_2)^4 - 12(\Delta_2)^2\Delta_3 + \frac{35}{8}(\Delta_2)^2\alpha_2 + \frac{5}{3}(\Delta_3)^2 + \frac{7}{48}(\alpha_2)^2 - \Delta_3\alpha_2, \\
&\quad + \frac{5}{2}\Delta_2\Delta_4 - \frac{3}{2}\Delta_2\dot{\alpha}_2 - \frac{1}{5}\Delta_5 + \frac{3}{20}\ddot{\alpha}_2 - \frac{1}{30}\alpha_3,
\end{aligned} \tag{65}$$

which is derived in the following subsection. The result is obtained as Eq. (38).

A.2. Expansion of τ in powers of ϵ

Let us derive Eq. (65). The retarded time τ depends on ϵ according to

$$\begin{aligned} 0 &= (z(\tau) - x) \cdot (z(\tau) - x) \\ &= (z(\tau) - z(0) - n\epsilon) \cdot (z(\tau) - z(0) - n\epsilon). \end{aligned} \quad (66)$$

For given x^μ , two solutions of Eq. (66) with respect to τ are possible. Of course, we select the solution with $\tau < 0$. It is found that, if one fixes n^μ in Eq. (66), τ is a function of ϵ . Expanding $(z(\tau) - x) \cdot (z(\tau) - x)$ in τ by using Eq. (60), we get

$$\begin{aligned} 0 &= (z(\tau) - x) \cdot (z(\tau) - x) \\ &= -\epsilon^2 + [1 - \Delta_2\epsilon]\tau^2 - \frac{1}{3}\Delta_3\epsilon\tau^3 - \frac{1}{12}[\alpha_2 + \Delta_4\epsilon]\tau^4 - \left[\frac{1}{24}\dot{\alpha}_2 + \frac{1}{60}\Delta_5\epsilon \right] \tau^5 \\ &\quad + \left[-\frac{1}{80}\ddot{\alpha}_2 + \frac{1}{360}\alpha_3 \right] \tau^6 + O(\epsilon^7). \end{aligned} \quad (67)$$

By using the relation $\tau = -\epsilon + O(\epsilon^2)$, the evaluation of $(z(\tau) - x) \cdot (z(\tau) - x)$ to order $O(\epsilon^4)$ gives

$$0 = -\epsilon^2 + [1 - \Delta_2\epsilon]\tau^2 + \frac{1}{3}\Delta_3\epsilon^4 - \frac{1}{12}\alpha_2\epsilon^4 + O(\epsilon^5), \quad (68)$$

so that we have

$$\tau^2 = [1 - \Delta_2\epsilon]^{-1} \left[\epsilon^2 - \frac{1}{3}\Delta_3\epsilon^4 + \frac{1}{12}\alpha_2\epsilon^4 \right] + O(\epsilon^5), \quad (69)$$

$$\tau = [1 - \Delta_2\epsilon]^{-1/2} \left[-\epsilon + \frac{1}{6}\Delta_3\epsilon^3 - \frac{1}{24}\alpha_2\epsilon^3 \right] + O(\epsilon^4). \quad (70)$$

From these, it follows that

$$\begin{aligned} \tau^3 &= -\epsilon^3 - \frac{3}{2}\Delta_2\epsilon^4 - \frac{15}{8}(\Delta_2)^2\epsilon^5 + \frac{1}{2}\Delta_3\epsilon^5 - \frac{1}{8}\alpha_2\epsilon^5 + O(\epsilon^6), \\ \tau^4 &= \epsilon^4 + 2\Delta_2\epsilon^5 + 3(\Delta_2)^2\epsilon^6 - \frac{2}{3}\Delta_3\epsilon^6 + \frac{1}{6}\alpha_2\epsilon^6 + O(\epsilon^7), \\ \tau^5 &= -\epsilon^5 - \frac{5}{2}\Delta_2\epsilon^6 + O(\epsilon^7). \end{aligned} \quad (71)$$

Using the expansion (71), the relation $\tau^6 = \epsilon^6 + O(\epsilon^7)$, and the expansion of $[1 - \Delta_2\epsilon]^{-1}$ with ϵ , we get from Eq. (67)

$$\tau^2 = \epsilon^2 + \Delta_2\epsilon^3 + \left[(\Delta_2)^2 - \frac{1}{3}\Delta_3 + \frac{1}{12}\alpha_2 \right] \epsilon^4$$

$$\begin{aligned}
& + \left[(\Delta_2)^3 - \frac{5}{6}\Delta_2\Delta_3 + \frac{1}{4}\Delta_2\alpha_2 + \frac{1}{12}\Delta_4 - \frac{1}{24}\dot{\alpha}_2 \right] \epsilon^5 \\
& + \left[(\Delta_2)^4 - \frac{35}{24}(\Delta_2)^2\Delta_3 + \frac{1}{2}(\Delta_2)^2\alpha_2 + \frac{1}{4}\Delta_2\Delta_4 - \frac{7}{48}\Delta_2\dot{\alpha}_2 \right. \\
& \quad \left. + \frac{1}{6}(\Delta_3)^2 - \frac{7}{72}\Delta_3\alpha_2 + \frac{1}{72}(\alpha_2)^2 - \frac{1}{60}\Delta_5 + \frac{1}{80}\ddot{\alpha}_2 - \frac{1}{360}\alpha_3 \right] \epsilon^6 \\
& + O(\epsilon^7).
\end{aligned} \tag{72}$$

From this, we obtain Eq. (65).

Appendix B

—A Trial Calculation—

In this appendix, We outline our trial calculation of the self-stress in terms of the retarded proper time mentioned in section5. The retarded time τ in Eq. (34) depends on the reference point x of the field. Now let us consider an extension $f_{\mu\nu}(x, \tau)$ of the function $F_{\mu\nu, \text{ret}}(x)$ over the time region beyond the fixed retarded time. We could consider arbitrary extensions of $f_{\mu\nu}(x, \tau)$ except in the region fixed by the retarded time. For the moment we choose the extended form

$$f_{\mu\nu}(x, \tau) = \frac{e}{\dot{z}(\tau) \cdot (z(\tau) - x)} \frac{d}{d\tau} \frac{\dot{z}(\tau)_\mu (z(\tau)_\nu - x_\nu) - \dot{z}(\tau)_\nu (z(\tau)_\mu - x_\mu)}{\dot{z}(\tau) \cdot (z(\tau) - x)}. \tag{73}$$

Let us evaluate the retarded field at the position of the point charge by

$$\lim_{\tau \rightarrow \tau_0} f_{\mu\nu}(z(\tau_0), \tau). \tag{74}$$

Now we set $\tau_0 = 0$ without loss of generality. By using Eqs. (60) and (61), we obtain the expansion of $f_{\mu\nu}(z(0), \tau)$ in powers of τ around $\tau = 0$:

$$\begin{aligned}
e^{-1} f_{\mu\nu}(z(0), \tau) &= \frac{1}{2}(2)_\mu(1)_\nu \tau^{-1} + \frac{2}{3}(3)_\mu(1)_\nu + \frac{3}{8}(4)_\mu(1)_\nu \tau + \frac{1}{4}(3)_\mu(2)_\nu \tau \\
&\quad + \frac{1}{3}\alpha_2(2)_\mu(1)_\nu \tau - (\mu \leftrightarrow \nu) + O(\tau^2).
\end{aligned} \tag{75}$$

(If we choose an extension other than Eq. (73), we obtain a different form of the expansion.) One should note that the term of order $O(\tau^{-2})$ disappears. This equation is similar to Eq. (41), so one could interpret that the first term of Eq. (75) contributes to the infinite Coulomb mass of the point charge. (Because $\tau < 0$, the sign of this term is equal to the sign of the first term of Eq. (41).) The second term also reproduces the radiation reaction force of the Lorentz-Dirac equation.

Let us calculate the zero-th terms of the expansion of $F_{\mu\alpha}F_\nu^\alpha$ in powers of τ by using Eq. (75). We can choose $f_{\mu\alpha}(z(0), \lambda\tau)f_\nu^\alpha(z(0), -\lambda\tau)$ (λ is an arbitrary positive

constant) or $f_{\mu\alpha}(z(0), \lambda\tau)f_{\nu}^{\alpha}(z(0), \lambda\tau)$ or any other form which is symmetric with respect to the exchange of the indices μ and ν . Although the evaluation of the zero-th terms of expansion does not depend on the value of λ , the spectrum is affected by λ , as we see later in an explicit calculation.

We now choose $f_{\mu\alpha}(z(0), \lambda\tau)f_{\nu}^{\alpha}(z(0), -\lambda\tau)$. The zero-th terms of the expansions are

$$\begin{aligned} e^{-2}(f_{\mu\alpha}(z(0), \lambda\tau)f_{\nu}^{\alpha}(z(0), -\lambda\tau))_0 &= \frac{11}{32}\dot{\alpha}_2(2)_{\mu}(1)_{\nu} - \frac{41}{72}\alpha_2(3)_{\mu}(1)_{\nu} + \frac{3}{16}(4)_{\mu}(2)_{\nu} \\ &+ \left[\frac{3}{32}\ddot{\alpha}_2 - \frac{59}{144}\alpha_3 + \frac{1}{6}(\alpha_2)^2 \right] (1)_{\mu}(1)_{\nu} + \frac{7}{24}\alpha_2(2)_{\mu}(2)_{\nu} - \frac{2}{9}(3)_{\mu}(3)_{\nu} + (\mu \leftrightarrow \nu), \end{aligned} \quad (76)$$

and

$$e^{-2}(f_{\mu\nu}(z(0), \lambda\tau)f^{\nu\mu}(z(0), -\lambda\tau))_0 = \frac{3}{8}\ddot{\alpha}_2 - \frac{59}{36}\alpha_3 + \frac{37}{18}(\alpha_2)^2. \quad (77)$$

For uniform acceleration, we have

$$e^{-2}a^{-4}(f_{\mu\alpha}(z(0), \lambda\tau)f_{\nu}^{\alpha}(z(0), -\lambda\tau))_0 = -\frac{5}{24}m_{\mu}m_{\nu} + \frac{5}{24}(1)_{\mu}(1)_{\nu}, \quad (78)$$

and

$$e^{-2}a^{-4}(f_{\mu\nu}(z(0), \lambda\tau)f^{\nu\mu}(z(0), -\lambda\tau))_0 = \frac{5}{12}. \quad (79)$$

Then we obtain the zero-th term of the expansion of stress-energy tensor in powers of τ :

$$T^{\mu\nu} = \pi\alpha \cdot \frac{5}{192\pi^2} \frac{\hbar a^4}{c^7} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (80)$$

For circular motion, we have

$$\begin{aligned} \frac{(f_{\mu\alpha}(z(0), \lambda\tau)f_{\nu}^{\alpha}(z(0), -\lambda\tau))_0}{e^2\gamma^8\Omega^4v^2} &= \frac{1}{24}[-14 + 5\gamma^{-2}]l_{1\mu}l_{1\nu} \\ &+ \frac{1}{72}[74 - 15\gamma^{-2}](1)_{\mu}(1)_{\nu} - \frac{1}{8}v[l_{2\mu}(1)_{\nu} + (1)_{\mu}l_{2\nu}] - \frac{4}{9}l_{2\mu}l_{2\nu}, \end{aligned} \quad (81)$$

and

$$\frac{(f_{\mu\nu}(z(0), \lambda\tau)f^{\nu\mu}(z(0), -\lambda\tau))_0}{e^2\gamma^8\Omega^4} = \frac{v^4}{36}[74 - 15\gamma^{-2}]. \quad (82)$$

Then, we obtain the zero-th term of the expansion of the stress-energy tensor in powers of τ

$$T^{\mu\nu} = \pi\alpha \cdot \frac{1}{576\pi^2} \frac{\hbar\gamma^8\Omega^4 v^2}{c^5} \times \begin{bmatrix} 74 - 15\gamma^{-2} & 0 & -18v/c & 0 \\ 0 & -10 + 15\gamma^{-2} & 0 & 0 \\ -18v/c & 0 & 10 - 15\gamma^{-2} & 0 \\ 0 & 0 & 0 & 74 - 15\gamma^{-2} \end{bmatrix}. \quad (83)$$

Thus we have obtained stress-energy tensors which are roughly the same order as the vacuum stresses multiplied by $\pi\alpha$. (Although the flux in Eq. (83) is proportional to v^3 , one finds that the degrees of the parameter representation of this flux is equal to that of vacuum stress if one takes $v^2 = 1 - \gamma^{-2}$ into account.)

We note that $F_{\mu\nu}F^{\nu\mu}$ of Eq. (77) is not zero in general motion, but $F_{\mu\nu}F^{\nu\mu}$ of Eq. (7) is precisely zero in any motion of the observer. Furthermore, we should note that, while the vacuum expectation value of $F_{\mu\alpha}F_\nu^\alpha$ includes the $\eta_{\mu\nu}$ term (see Eq. (6)), $F_{\mu\alpha}F_\nu^\alpha$ calculated in this Appendix (see Eq. (76)) does not include $\eta_{\mu\nu}$ explicitly. Because of this fact, $F_{1\alpha}F_1^\alpha$ and $F_{2\alpha}F_2^\alpha$ are zero in the case of uniform acceleration in the derivation in this Appendix, in contrast to the case of the vacuum stress. We note that $F_{1\alpha}F_1^\alpha$ and $F_{2\alpha}F_2^\alpha$ in the case of uniform acceleration evaluated in section 4 (Eq. (50)) have small, but nonzero values which come from the *angular average* of the $n^\mu n^\nu$ term in Eq. (46). Therefore, the angular average method of section 4 may induce an $\eta_{\mu\nu}$ -like contribution when one evaluates $F_{\mu\alpha}F_\nu^\alpha$.

Next let us calculate the spectrum of the self-stress in the case of uniform acceleration. We have

$$T^{\mu\nu} = \lim_{\tau \rightarrow 0} \frac{f^{03}(z(0), \lambda\tau) f^{03}(z(0), -\lambda\tau)}{8\pi} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (84)$$

Thus all we have to do is to calculate the Fourier transform of $f^{03}(z(0), \lambda\tau) f^{03}(z(0), -\lambda\tau)$. We define

$$G(\tau) = e^{-2} f^{03}(z(0), \tau) f^{03}(z(0), -\tau). \quad (85)$$

Then it follows that

$$\begin{aligned} \lim_{\tau \rightarrow 0} G(\lambda\tau) &= \int_{-\infty}^{\infty} d\tau \delta(\tau) G(\lambda\tau) \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\omega e^{i\omega\tau} G(\lambda\tau) \\ &= \frac{1}{2\pi} \int_0^{\infty} d\omega \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} [G(\lambda\tau) + G(-\lambda\tau)]. \end{aligned} \quad (86)$$

$G(\tau)$ has a pole at $\tau = 0$. We now move this pole above the real axis of complex τ plane. Hence $G(\tau)$ takes the form

$$G(\tau) = -\frac{a^4[1 - \cosh(a\tau)]^2}{\sinh^6[a(\tau - i\epsilon)]}, \quad (87)$$

where ϵ is an infinitesimal positive number. This function has a periodicity $G(\tau + 2\pi a^{-1}i) = G(\tau)$. By using this property, we can easily perform the integration over τ in Eq. (86) (see section 4.4 of Ref. [6]). Although the calculation is similar to that of the Unruh effect, the residue of $G(\tau)$ at the pole $\tau = \pi i/a$ causes the result to take a rather awkward form. We obtain

$$\begin{aligned} \lim_{\tau \rightarrow 0} G(\lambda\tau) = \frac{1}{2\lambda^2} \int_0^\infty d\omega & \left[\frac{\omega a^2}{2} + \left(\frac{11}{10} + \frac{2}{3} \frac{\omega^2}{\lambda^2 a^2} + \frac{1}{15} \frac{\omega^4}{\lambda^4 a^4} \right) \frac{2\omega a^2}{e^{\pi\omega/\lambda a} - 1} \right. \\ & \left. - \left(\frac{3}{5} + \frac{2}{3} \frac{\omega^2}{\lambda^2 a^2} + \frac{1}{15} \frac{\omega^4}{\lambda^4 a^4} \right) \frac{2\omega a^2}{e^{2\pi\omega/\lambda a} - 1} \right]. \end{aligned} \quad (88)$$

In this spectrum, the Planckian terms $\omega^3(e^{\pi\omega/\lambda a} - 1)$ and $\omega^3(e^{2\pi\omega/\lambda a} - 1)$, which correspond to the temperatures $\lambda a/\pi$ and $\lambda a/2\pi$, respectively, appear. The first term, $\omega a^2/2$, of the spectrum resembles the term $\omega a^2/2$ in Eq. (16), which comes from the contribution of zero-point energy $\omega(\omega^2 + a^2)/2$. However, we cannot suggest with confidence that these results reflect some facts of real physics, because our evaluation here is, at present, rather artificial and awkward.

Finally, let us note the result of calculation in the case of $f_{\mu\alpha}(z(0), \lambda\tau) f_\nu^\alpha(z(0), \lambda\tau)$. We find that for uniform acceleration,

$$T^{\mu\nu} = \pi\alpha \cdot \frac{5}{192\pi^2} \frac{\hbar a^4}{c^7} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (89)$$

and for circular motion,

$$\begin{aligned} T^{\mu\nu} = \pi\alpha \cdot \frac{1}{576\pi^2} \frac{\hbar\gamma^8\Omega^4 v^2}{c^5} & \times \begin{bmatrix} -10 + 15\gamma^{-2} & 0 & 18v/c & 0 \\ 0 & 74 - 15\gamma^{-2} & 0 & 0 \\ 18v/c & 0 & -74 + 15\gamma^{-2} & 0 \\ 0 & 0 & 0 & -10 + 15\gamma^{-2} \end{bmatrix}. \end{aligned} \quad (90)$$

The spectrum of the stress-energy tensor in the case of uniform acceleration is just the opposite of Eq. (88).

References

- [1] S. A. Fulling, Phys. Rev. **D7**, (1973) 2850.
- [2] P. C. W. Davies, J. Phys. **A8**, (1975) 609.
- [3] W. G. Unruh, Phys. Rev. **D14**, (1976) 870.
- [4] B. S. DeWitt, in *General relativity*, ed. S. W. Hawking and W. Israel (Cambridge University Press, 1979), p.680.
- [5] N. D. Birrell and P. C. W. Davies, *Quantum Fields in curved space*, section 3.3 (Cambridge University Press, 1982).
- [6] S. Takagi, Prog. Theor. Phys. Supp. **88** (1986).
- [7] J. R. Letaw and J. D. Pfautsch, Phys. Rev. **D22**, (1980) 1345.
- [8] J. S. Bell and J. M. Leinaas, Nucl. Phys. **B212**, (1983) 131; **B284**, (1987) 488.
- [9] D. W. Sciama, P. Candelas and D. Deutsch, Adv. Phys. **30**, (1981) 327.
- [10] S. R. Mane, Phys. Rev. **D43**, (1991) 3578.
- [11] H. C. Rosu, Report No. IFUG-28-94, gr-qc/9412033.
- [12] A. Higuchi, G. E. A. Matsas and D. Sudarsky, Phys. Rev **D46**, (1992) 3450.
- [13] A. Higuchi and G. E. A. Matsas, Phys. Rev. **D48**, (1993) 689.
- [14] T. Fulton and F. Rohrlich, Ann. Phys. (N.Y.) **9**, (1960) 499.
- [15] D. G. Boulware, Ann. Phys. (N.Y.) **124**, (1980) 169.
- [16] F. Rohrlich, *Classical Charged Particles* (Addison-Wesley, 1990), section 5-3.
- [17] P. W. Milonni, *The Quantum Vacuum* (Academic Press, 1994).
- [18] For implication of the Unruh effect in the classical theory, for example, see Ref. [13] and K. Srinivasan, L. Sriramkumar and T. Padmanabhan, Phys. Rev. **D56**, (1997) 6692; Int. J. Mod. Phys. **D6**, (1997) 607.
- [19] S. Hacyan, Phys. Rev. **D32**, (1985) 3216.
- [20] S. Hacyan and A. Sarmiento, Phys. Rev. **D40**, (1989) 2641.
- [21] Our aim is to find out the trace of the Unruh effect in the calculation involving the self-field of the charged particle. In this connection, the following paper would be interesting: A. O. Barut and J. P. Dowling, Phys. Rev. **A41**, (1990) 2277.

- [22] C. Teitelboim, Phys. Rev. **D4**, (1971) 345.
- [23] P. A. M. Dirac, Proc. Roy. Soc. (London) **A167**, (1938) 148.
- [24] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1970).